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The Self-Contradictory Foundations of Formalistic Quantum Measurement Theories†

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Abstract

One school of thought in quantum measurement theory adopts as its aim the derivation of a certain mixed statistical operator to characterize the ensemble of global object-apparatus systems subsequent to the measurement interaction. This paper demonstrates that even if that goal were achieved, the consequent theory of measurement would be self-contradictory; hence the measurement problem is improperly formulated. The epistemological root of the difficulty is discussed briefly. A logical resolution is offered in terms of quantum axiomatics by emphasizing the actual relationship of quantum theory to experimental and observational data.

1. *Formal Quantum Measurement Theory and Its Elusive Goal*

Whenever quantum mechanics is used to describe a physical experience of macroscopic scope, strenuous theoretical efforts are often made to cast the irreducibly probabilistic and indeterministic theory into the prosaic mechanical molds of the classical world view. The results are seldom gratifying, for they frequently reflect a retrogressive tendency in the philosophy of physics, where positive epistemological innovation aimed at comprehending macroexperiences in terms of *sui generis* quantal constructs would be far more satisfying.

The quantum mechanics of macrosystems embraces several basic dilemmas in the foundations of physics, among these the classical limit problem, the problem of irreversibility, and the theory of measurement. It is the latter with which the present analysis is concerned. 'Quantum theory of measurement' is a rubric which encompasses a variety of theoretical investigations,‡ ranging in methodology and content from metaphysical and epistemological studies of quantum physics to formal manipulations

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‡ For extensive bibliographies, cf. d'Espagnat (1965) and Jammer (1966). Recent contributions are cited in Park (1968), Fine (1970), and Moldauer (1972).

of the quantal algorithm sometimes devoid of any clear empirical interpretation. The subject is controversial, and lucid contributions to the field can be quite stimulating. Indeed, at the core of the many arguments over quantal measurement there is an important philosophical problem. It is a problem of explication (Park, 1968), of transposition of the old concept of measurement into a new distinctly quantal context, of evolution toward a modern *Weltansicht* for physics in which the mathematical models of quantum theory could replace, even in descriptions of macroscopic apparatus, the mechanical models of our present intuition.

Unfortunately, this significant problem in the philosophical foundations of physics is almost traditionally oversimplified in the following formal manner. The focus is usually placed upon the quantal characterization of the 'pointers' which record, for direct perception by the experimenter, the data measured using an apparatus. Let 'pointer position' be an observable A of the apparatus (hence of the global system comprised of object system plus apparatus) and suppose that the measurement interaction is such that afterwards the probability distribution for pointer positions is nonzero only for a specific subset $\{a_n\}$ of A-eigenvalues. (For the contemplated interaction to be a measurement, there must of course be an appropriate correlation between the A-eigenvalues and the eigenvalues of an observable of interest in the object system.)

An early goal of the quantum theory of measurement was to prove this proposition:

(a) The postmeasurement state of the apparatus M alone must be the (reduced) statistical operator

$$\rho_M = \sum p_n |\psi_n\rangle\langle\psi_n| \tag{1.1}$$

where ψ_n denotes an A-eigenvector belonging to a_n. The problem was first studied by von Neumann (1955), who demonstrated that certain measurement interactions could generate correlations leading to (1.1) as the postmeasurement state of the apparatus alone. Later Wigner (1952) and others (Araki & Yanase, 1961) showed that in realistic cases (a) is more likely to be valid approximately than rigorously. For philosophical reasons to be examined below, other theorists have insisted upon the following stronger assertion as a necessary trait of the measurement process:

(b) The postmeasurement state of the global system (object-plus-apparatus) must be the statistical operator

$$\rho = \sum p_n |\Psi_n\rangle\langle\Psi_n| \tag{1.2}$$

where Ψ_n denotes an eigenvector belonging to eigenvalue a_n of the global system observable $1 \otimes A$. (1 is the identity in the object's Hilbert space.)

Attempts to establish (b), which I shall call the Formal Quantum Theory of Measurement (FQTM) have recently culminated in the work of Fine (1970), who showed, under rather general conditions, that (1.2) cannot be the postmeasurement global state. Soon there was a rejoinder from Moldauer (1972), who recalled von Neumann's proof that (1.1) can be the postmeasurement apparatus state; i.e., ρ_M can be the reduced density operator for the apparatus even when the global system (object-plus-apparatus) is in a pure state expressible in terms of the $\{\Psi_n\}$ only as a coherent superposition.

Such an exchange, in which the participants do not speak directly to each other's points, is nothing new to the quantum theory of measurement. Undoubtedly the school of thought represented in this instance by Fine remains adamant in its belief that quantum physics is beset with a grave problem due to the elusive character of a proof for proposition (b).

Having granted earlier that explication of the measurement process is a significant facet of quantal foundations research, I shall explain below why the proof or disproof of proposition (b) is essentially irrelevant. There do exist profound physical and philosophical problems concerning the adaptation of quantum mechanical modes of thought to the realm of ordinary macroscopic observation; but FQTM offers only a misleading and contradictory rendition of the problem of measurement.

2. *The Inevitable Failure of FQTM*

Let us suppose for the moment that the goal of the formal measurement theorists had been achieved; i.e., assume there does exist an initial apparatus state and a correlating interaction between object and apparatus such that the post-measurement global state is given by (1.2). To consider a simple case, let the observable being measured be dichotomic so that after the measurement process the experimenter would find the pointer position to be either a_1 or a_2. According to our hypothetical FQTM, the post-measurement global state is therefore

$$\rho = p|\Psi_1\rangle\langle\Psi_1| + (1-p)|\Psi_2\rangle\langle\Psi_2| \tag{2.1}$$

What has been accomplished? What does (2.1) mean physically? Conventional FQTM wisdom interprets (2.1) as a description of an ensemble of object-apparatus systems, of which the fraction p is in $1 \otimes A$-eigenstate Ψ_1 and certain therefore to 'possess' pointer position a_1. The rest of the ensemble members are said to be in state Ψ_2, hence to have pointer position a_2. This is to be contrasted to a global pure state such as

$$\Phi_f = p^{1/2}\Psi_1 + e^{if}(1-p)^{1/2}\Psi_2 \tag{2.2}$$

which FQTM proponents regard as a description in which the pointer on the apparatus, contrary to macrophysical experience, has no position at all. We shall return to this point later.

First, however, the seeming reasonableness of the FQTM interpretation of (2.1) bears closer scrutiny. If, as we have assumed for the sake of argument, FQTM were in fact successful in deriving (2.1), would this actually mean that for each run of an experiment, the object-apparatus system would invariably emerge either in the state Ψ_1 or in the state Ψ_2?

A simple application of a theorem due to Schrödinger (1936) establishes beyond doubt that the foregoing question must be answered in the negative. Hence even if the FQTM school succeeded in proving proposition (b), it still would have failed to confer upon the postmeasurement global system a state whose physical interpretation satisfied the philosophical demands that motivated the FQTM effort in the first place.

The insurmountable difficulty for FQTM may be expressed as follows. If the ensemble characterized by (2.1) can, as in the FQTM interpretation, be regarded as consisting of two pure subensembles with state vectors Ψ_1, Ψ_2, it can, with equally valid justification, be said to be consist of several pure subensembles, one of which has a state vector Φ_1 which can be *arbitrarily preassigned*, provided only that Φ_1 be a superposition of Ψ_1, Ψ_2:

$$\Phi_1 = c_{11}\,\Psi_1 + c_{12}\,\Psi_2 \tag{2.3}$$

For an explicit example, consider the following equality:

$$\begin{aligned} \rho &= p|\Psi_1\rangle\langle\Psi_1| + (1-p)|\Psi_2\rangle\langle\Psi_2| \\ &= w|\Phi_1\rangle\langle\Phi_1| + (1-w)|\Phi_2\rangle\langle\Phi_2| \end{aligned} \tag{2.4}$$

provided

$$w = \left(\frac{|c_{11}|^2}{p} + \frac{|c_{12}|^2}{1-p}\right)^{-1} \tag{2.5}$$

$$\Phi_2 = c_{21}\,\Psi_1 + c_{22}\,\Psi_2 \tag{2.6}$$

$$c_{21} = \left(\frac{wp}{(1-w)(1-p)}\right)^{1/2} |c_{12}| \tag{2.7}$$

$$c_{22} = \frac{p-1}{p}\left(\frac{c_{11}}{c_{12}}\right)^{*} c_{21} \tag{2.8}$$

Equations (2.4)–(2.8) were obtained by applying Schrödinger's general work on mixtures to the special case (2.1); however, it is a straightforward matter to verify the result (2.4) directly.

If Ψ_1 and Ψ_2 are orthogonal, then in general Φ_1 and Φ_2 will not be orthogonal unless $p = \frac{1}{2}$. Although textbook introductions to the statistical operator commonly use, for mathematical simplicity, only the spectral expansion of ρ, there is no physical reason for such a restriction. Indeed if two pure ensembles having known nonorthogonal state vectors were combined, the statistical operator for the new total ensemble would be immediately and naturally expressible as a linear combination of nonorthogonal projectors. Thus the possible nonorthogonality of Φ_1 and Φ_2 in no way vitiates our argument against FQTM. In fact, the FQTM assumption that Ψ_1 and Ψ_2 must be eigenstates of $1 \otimes A$, hence orthogonal, is perhaps too strong; for example, two minimum uncertainty wave packets centered at a_1, a_2 would seem to be a reasonable quantal description of pointer positions, yet they would be only approximately orthogonal.

To return now to the main point, let us choose as Φ_1 the state vector Φ_f defined in (2.2); i.e., let

$$c_{11} = p^{1/2}, \qquad c_{12} = (1-p)^{1/2}\,\mathrm{e}^{if} \tag{2.9}$$

From (2.5)–(2.8), we then obtain

$$w = \tfrac{1}{2}, \qquad c_{21} = p^{1/2}, \qquad c_{22} = -(1-p)^{1/2}\,\mathrm{e}^{if} \tag{2.10}$$

and therefore

$$\begin{aligned} \rho &= p|\Psi_1\rangle\langle\Psi_1| + (1-p)|\Psi_2\rangle\langle\Psi_2| \\ &= \tfrac{1}{2}|\Phi_1\rangle\langle\Phi_1| + \tfrac{1}{2}|\Phi_2\rangle\langle\Phi_2| \end{aligned} \tag{2.11}$$

where

$$\Phi_1 = p^{1/2}\,\Psi_1 + \mathrm{e}^{if}(1-p)^{1/2}\,\Psi_2 = \Phi_f \tag{2.12}$$

and

$$\Phi_2 = p^{1/2}\,\Psi_1 - \mathrm{e}^{if}(1-p)^{1/2}\,\Psi_2 = \Phi_{f+\pi} \tag{2.13}$$

Global state vectors of the general type Φ_f inevitably emerge dynamically from measurement processes in which object and apparatus have been initially in pure states. As noted above, such superpositions of pointer position eigenstates are anathema to the FQTM school, who insist that Φ_f means in quantum theory that the pointer has no position. Ironically, we see now that even if the FQTM program were successful in replacing Φ_f by a statistical mixture (2.1) of pointer position eigenstates, that very mixture (2.11) could also be partitioned into pure subensembles with state vectors Φ_f, $\Phi_{f+\pi}$, *both* of the kind whose alleged unacceptability originally inspired the FQTM investigations.

To use the philosophical language of FQTM, the ensemble in which the fraction p of systems has state Ψ_1 and pointer position a_1 with the remainder having state Ψ_2 and pointer position a_2 is also an ensemble in which half the systems have state Φ_f and no pointer position at all and the other half have state $\Phi_{f+\pi}$ and similarly no pointer position at all. Thus each system has at once a definite pointer position and no pointer position! *Reductio ad absurdum.* Even if FQTM were successful, it would be self-contradictory.

3. *Logical and Epistemological Aspects of the Problem*

The paradox derived above undermines the basic structure of FQTM. It does not, however, expose any logical defect in quantum physics itself. Quantum mechanics is obviously an established physical theory whose epistemic connections to nature are well known; it is applied routinely and uncontroversially to myriad physical situations. The flaw in FQTM is at root philosophical, inhering in a steadfast refusal to accept quantum mechanics for what it is, a theory about the statistics of measurement results. Instead FQTM theorists are really pretending that quantum mechanics is epistemologically like classical analytical dynamics, which is traditionally construed to be a theory dealing primarily with the intrinsic properties of physical objects.

This is not the place to dwell at length upon the epistemological subtleties which distinguish the FQTM school from quantum physicists like the present author who are untroubled by the apparent impossibility of deriving proposition (b). Fundamentally, FQTM appears to be founded on a quantal version of epistemological realism in which every system is presumed to possess a state vector as an intrinsic attribute; thus the pointer must 'have' either Ψ_1 or Ψ_2, the mixture (2.1) being interpreted to mean that some members of the ensemble have property Ψ_1, others have property Ψ_2. The contrary philosophical view, in which FQTM becomes an irrelevant quest and its internal contradictions unimportant, is a modern version of epistemological idealism wherein the function of physical theory is understood to be limited to describing experience rather than innate properties of objects. Probably the most incisive treatment of the latter view has been given by Margenau in his general scientific epistemology (Margenau, 1950) and in his latency theory of quantal measurement (Park & Margenau, 1968).

However, instead of pursuing these deeply epistemological matters any further, it seems more appropriate for the present analysis to set forth in terms of quantum axiomatics the root of the FQTM difficulty and its resolution. From the perspective of FQTM, quantum mechanics is based upon this postulate (among others not in contention): each (closed) system possesses a state vector ψ. The scalar product $\langle\phi_n|\psi\rangle$ where ϕ_n is the eigenvector belonging to eigenvalue f_n of an observable F, is then interpreted as the probability amplitude that upon F-measurement the system will execute a quantum jump from state ψ to state ϕ_n, f_n being the numerical datum recorded by the experimenter. Only if $\psi = \phi_n$ is f_n the certain result of the F-measurement; hence in that case the system is said to have the F-value f_n.

It is in this axiomatic framework that the FQTM problem germinates. Because it is intuitively felt that pointers *have* positions, naturally it must be demanded that pointers *have* the corresponding state vectors as properties. As we have seen, this viewpoint ultimately leads to self-contradiction and paradox.

If we abandon the notion that each system *possesses* a state vector, the need for FQTM dissolves. (Such an abandonment is an easy step indeed for the experimental scientist who gathers numerical data, not Hilbert vectors.) The axiom which endowed each system with a state vector can be replaced by the following postulate: every repeatable preparation is characterized by a statistical operator ρ in the sense that $\mathrm{Tr}(\rho F)$ is the arithmetic mean of F-data collected by performing F-measurements upon the members of an ensemble of systems each *prepared* identically (in the manner Π). A preparation Π is a set of reproducible empirical operations undertaken prior to each run of an experiment. The concept of preparation (Margenau, 1937, 1963) is just as important to the fabric of quantum theory as the more controversial concept of measurement, but until recently remarkably little attention has been paid to the quantum theory of preparation (Band & Park, 1972).

Once it is clearly understood that quantum theory exists to regularize and to make predictions concerning statistical quantities associated with collectives of data, not to describe as an unseen world of abstract saltatorial vectors borne by picturable mechanical objects, all of the quantal assertions which inspired FQTM and its attendant problems turn out to be merely colloquialisms automatically voiced but surely not taken literally by physical scientists. For example (Park, 1970), $\langle\phi_n|\psi\rangle$ is not the probability amplitude that 'the state will jump to ϕ_n when F is measured', nor even the amplitude that 'the system will be found in state ϕ_n'; such language is devoid of experiential meaning. The scalar product $\langle\phi_n|\psi\rangle$ is the probability amplitude for the emergence of the F-datum f_n from an F-measurement act performed on a system prepared in the manner characterized by $\rho = |\psi\rangle\langle\psi|$.

Moreover, a preparation characterized by a mixed statistical operator like (2.1) cannot be rationally interpreted to mean that the ensemble is constituted of some systems possessing Ψ_1, others possessing Ψ_2; the nonuniqueness illustrated by (2.11) demolishes that simplistic view of mixtures. To clarify properly the physical distinction between pure and mixed preparations, von Neumann's original conception of ensemble homogeneity must be recalled.

A pure or homogeneous ensemble is one that cannot be partitioned into physically distinct subensembles; i.e., given ρ, there do not exist distinct statistical operators $\rho^{(1)}$, $\rho^{(2)}$ such that

$$\rho = w_1\rho^{(1)} + w_2\rho^{(2)} \tag{3.1}$$

According to von Neumann's mathematical analysis, every pure ensemble is characterized by a statistical operator of the form $\rho = |\psi\rangle\langle\psi|$. A mixture is then an inhomogeneous ensemble; its statistical operator can be resolved as in (3.1) with $\rho^{(1)}$, $\rho^{(2)}$ distinct.

This theoretical concept of partitioning ensembles obtains its empirical meaning in the process of *selection*. If an ensemble is mixed, then in principle there exist 'filtration' procedures wherein certain systems of the ensemble are rejected and the rest are then regarded as the new ensemble of interest. In terms of the preparation concept, the new ensemble is generated by subjecting systems to the original preparation procedure plus selection. For a pure ensemble, selection is in principle impossible.

The nonuniqueness of mixture resolutions into pure subensembles is now unproblematical, since we do not claim that each system possesses a (perhaps unknown) state vector. In fact the nonuniqueness merely affirms that a given mixture could have been produced by combining with appropriate weights entirely different sets of homogeneous preparations; conversely, the mixture can in principle be subdivided by selection in many different ways.

Finally, let us consider again the matter of pointer positions. There can be no quarrel with the empirical fact which inspired the FQTM quest, viz., that a macroscopic pointer in common observation seems to be endowed with classical kinematic 'properties' like position and velocity. However, we would insist that whether or not the pointer *possesses* those attributes even when it is not observed is an ontological question beyond the competence or interest of science. Thus for quantum mechanics to be compatible with human experience involving pointers, only the following proposition need be proved: after the measurement interaction between object and apparatus, the *ensemble of pointers* (as opposed to the ensemble of global object-apparatus systems) must be a mixture which can by the process of selection be partitioned into subensembles each of which has a statistical operator corresponding to a single macrokinematical state of the pointer. In other words, since an ensemble of actually observed pointers can in fact be described by saying that each individual pointer has a certain 'position' and 'velocity', quantum physics should not be in conflict with this common empirical experience. That indeed there is *in principle* no conflict was long ago demonstrated by von Neumann and others by proving the proposition (a) cited earlier. Today the interesting problems of measurement lie in the realm of clarification. To deepen our grasp of quantum theory as applied to macroscopic systems is an exciting philosophical challenge. It is best appreciated in full recognition that quantum mechanics deals primarily with data, with physical experience, and not with a neoclassical mechanical universe of objects possessing state vectors as properties.

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